Midterm Exam No. 02 (2021 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

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Date: Tuesday, 2021 Mar 30

1. (**20 points.**) The vector potential for a point magnetic dipole moment is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^3}. \tag{1}$$

The magnetic field due to a point magnetic dipole $\mathbf{m}$ at a distance $\mathbf{r}$ away from the magnetic dipole is given by the expression

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\left[3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{m}\right]}{r^3}, \qquad r\neq 0. \tag{2}$$

These expressions are for reference. This question, probably, can also be answered without relying on these expressions. Sketch the magnetic field lines due to two identical point magnetic dipole moments separated by a distance a and their dipole moments parallel to the line joining the two dipoles. Does the magnetic field go to zero anywhere? If yes, identify the points. If not, why not?

2. (**20 points.**) Determine the total magnetic dipole moment for the following configuration. The current in the loop is I and each fold in the loop is of length a.

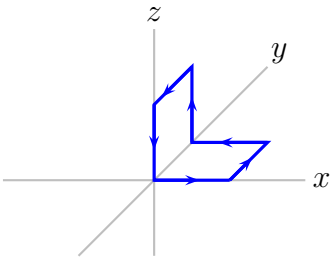

Figure 1: Problem 2

3. (**20 points.**) The complete elliptic integrals have the power series expansions

$$K(k) = \frac{\pi}{2}\sum_{n=0}^{\infty}\left[\frac{(2n)!}{2^{2n}(n!)^2}\right]^2 k^{2n} = \frac{\pi}{2}\left[1+\frac{1}{4}k^2+\frac{9}{64}k^4+\ldots\right], \tag{3a}$$

$$E(k) = \frac{\pi}{2}\sum_{n=0}^{\infty}\left[\frac{(2n)!}{2^{2n}(n!)^2}\right]^2 k^{2n} = \frac{\pi}{2}\left[1-\frac{1}{4}k^2-\frac{3}{64}k^4-\ldots\right]. \tag{3b}$$

The leading order contribution in the power series expansions are from $K(0)$ and $E(0)$. Evaluate the leading order contribution of

$$\left[K(k) - \frac{(2-k^2)}{2(1-k^2)}E(k)\right]. \tag{4}$$

Hint: Truncate all series expansions to order k^0 and collect the terms. If it is zero, repeat for order k^2. Repeat for subsequent higher orders until you obtain a non-zero contribution.

4. (**20 points.**) A hypothetical particle is observed by an inertial observer to be moving with non-uniform superluminal speed ($v > c$) at every instant of time from remote past to remote future. Draw a plausible world line of such a particle.