

Equation Sheet for PHYS-205A University Physics

(Last updated: January 16, 2023)

This list will evolve during the semester.

1. General mathematics:

(a) Units:

$$c = 10^{-2}, \quad m = 10^{-3}, \quad \mu = 10^{-6}, \quad n = 10^{-9}, \quad p = 10^{-12}. \quad (1a)$$

$$d = 10^2, \quad k = 10^3, \quad M = 10^6, \quad G = 10^9, \quad T = 10^{12}. \quad (1b)$$

(b) Geometry of a right triangle:

$$\sin \theta = \frac{\text{opp. to angle}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opp. to angle}}{\text{adj to angle}}, \quad (2a)$$

$$\cos \theta = \frac{\text{adj. to angle}}{\text{hypotenuse}}, \quad A^2 = A_x^2 + A_y^2. \quad (2b)$$

(c) Quadratic equation:

$$a x^2 + b x + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (3)$$

(d) Calculus:

$$\frac{d}{dx} x^n = n x^{n-1}, \quad \int x^n dx = \frac{x^{n+1}}{n+1}. \quad (4)$$

2. Kinematic equations:

(a) Velocity and acceleration:

$$v(t) = \frac{dx}{dt}, \quad a(t) = \frac{dv}{dt}. \quad (5)$$

(b) Uniform velocity ($a = 0$):

$$\Delta x = v \Delta t. \quad (6)$$

(c) Uniform acceleration:

$$v_f = v_i + a \Delta t; \quad \Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2; \quad v_f^2 = v_i^2 + 2 a \Delta x; \quad (7a)$$

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}; \quad \Delta x = v_f \Delta t - \frac{1}{2} a \Delta t^2. \quad (7b)$$

(d) Time of flight, horizontal range, and maximum height in projectile motion:

$$\Delta T = \frac{2v_0 \sin \theta_0}{g}, \quad R = \frac{v_0^2 \sin 2\theta_0}{g}, \quad H = \frac{v_0^2 \sin^2 \theta_0}{2g}. \quad (8)$$

(e) Relative velocity: $\vec{v}_{AB} = \vec{v}_{AG} + \vec{v}_{GB}$.

3. Forces:

(a) Newton's law:

$$\vec{F}_1 + \vec{F}_2 + \dots = m \vec{a} \quad (9)$$

(b) Gravitational force:

$$F_G = \frac{G m_1 m_2}{R^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (10)$$

(c) Force due to friction:

$$F_f \begin{cases} \leq \mu_s N, & \text{(static case),} \\ = \mu_k N, & \text{(kinetic case).} \end{cases} \quad (11)$$

(d) Circular motion:

$$v = \omega r, \quad \omega = 2\pi f, \quad f = \frac{1}{T}, \quad (12)$$

$$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r = \frac{4\pi^2}{T^2} r \quad (13)$$

(e) Resistive forces: $R = bv$ (for small speeds) and $R = \frac{1}{2} D \rho A v^2$ (for high speeds).

4. Work and energy:

(a) Kinetic energy:

$$K = \frac{1}{2} m v^2 \quad (14)$$

(b) Work done by a force:

$$W = \int \vec{F} \cdot d\vec{l} \rightarrow F d \cos \theta \quad (15)$$

(c) Work-kinetic energy theorem:

$$W_1 + W_2 + \dots = \Delta K \quad (16)$$

(d) Potential energies:

$$U_g = mgh, \quad U_s = \frac{1}{2} k x^2. \quad (17)$$

(e) Mechanical energy:

$$\Delta K + \Delta U_g + \Delta U_s = W_{\text{fric}} + \dots \quad (18)$$

5. Linear momentum:

$$\vec{p} = m\vec{v}, \quad \vec{p}_f - \vec{p}_i = \int_i^f \vec{F} dt. \quad (19)$$

(a) Conservation of linear momentum:

$$\vec{p}_{1i} + \vec{p}_{2i} + \dots = \vec{p}_{1f} + \vec{p}_{2f} + \dots \quad (20)$$

(b) Elastic collision of two objects in one dimensional motion:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}, \quad (21a)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}. \quad (21b)$$

(c) Center of mass:

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots} \rightarrow \frac{\int x dm}{\int dm}. \quad (22)$$

6. Rotational dynamics:

(a) Kinematic equations:

i. Constant angular speed ($\alpha = 0$): $\Delta\theta = \omega\Delta t$.

ii. Constant angular acceleration:

$$\omega_f = \omega_i + \alpha \Delta t, \quad \Delta\theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2, \quad (23)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta, \quad \Delta\theta = \omega_f \Delta t - \frac{1}{2} \alpha \Delta t^2, \quad \frac{\Delta\theta}{\Delta t} = \frac{\omega_i + \omega_f}{2}. \quad (24)$$

(b) Rotational inertia (moment of inertia): $I = \int r^2 dm$.

$$I = \begin{cases} MR^2, & \text{Point mass, distance } R \text{ from axis,} \\ MR^2, & \text{Circular ring, about symmetry axis of ring,} \\ \frac{2}{3}MR^2, & \text{Spherical shell, about diameter,} \\ \frac{1}{2}MR^2, & \text{Solid cylinder, about symmetry axis of cylinder,} \\ \frac{2}{5}MR^2, & \text{Solid sphere, about diameter.} \end{cases} \quad (25a)$$

(c) Torque:

$$\tau = RF \sin \theta, \quad \vec{\tau} = \frac{d\vec{L}}{dt}. \quad (26)$$

(d) Rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2. \quad (27)$$

(e) Angular momentum:

$$L = I\omega, \quad \vec{L} = \vec{r} \times \vec{p}. \quad (28)$$

7. Gravitation:

$$\vec{F} = -\hat{r} \frac{Gm_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (29)$$

$$U = -\frac{Gm_1 m_2}{r}. \quad (30)$$

8. Waves and oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}, \quad \lambda = \frac{2\pi}{k}, \quad v = \lambda f. \quad (31)$$

(a) Oscillations:

$$x = A \cos(\omega t + \delta) \quad (32)$$

(b) Simple pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}}. \quad (33)$$

(c) Mass-spring system:

$$T = 2\pi \sqrt{\frac{m}{k}}. \quad (34)$$

9. Fluid dynamics:

(a) Density-mass relation: (Density of water = 10^3 kg/m^3 .)

$$m = \rho V. \quad (35)$$

(b) Pressure:

$$P = \frac{F}{A}. \quad (36)$$

(c) Pressure in a static fluid:

$$P_2 = P_1 + \rho h g. \quad (37)$$

$$1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2.$$

(d) Buoyant force:

$$B = m_{\text{dis}} g, \quad (38)$$

where $m_{\text{dis}} = \rho_{\text{dis}} V_{\text{dis}}$ is the mass of the displaced liquid.

(e) Continuity equation:

$$A_1 v_1 = A_2 v_2. \quad (39)$$

(f) Bernoulli's equation:

$$P + \rho h g + \frac{1}{2} \rho v^2 = \text{const} \quad (40)$$