Midterm Exam No. 01 (2023 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

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Date: 2023 Feb 23

1. (**20 points.**) For a constant vector $\mathbf{p}$, (without invoking the Maxwell equations,) evaluate

$$\nabla^2 \left(\frac{\mathbf{p} \cdot \mathbf{r}}{r^3} \right) \tag{1}$$

for $r \neq 0$.
Hints: For insight, recall that the electric potential of a point dipole $\mathbf{p}$ placed at the origin is

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p} \cdot \mathbf{r}}{r^3}. \tag{2}$$

2. (**20 points.**) A point charge Q sits at the center of an isolated conducting spherical shell of inner radius a and outer radius b. Using Gauss's law and symmetry arguments find the expression for the magnitude and direction of the electric field in regions $r < a$, $a < r < b$, and $b < r$.

3. (**20 points.**) The electromagnetic energy density U and the corresponding energy flux vector $\mathbf{S}$ are given by, ($\mathbf{D} = \varepsilon_0 \mathbf{E}$, $\mathbf{B} = \mu_0 \mathbf{H}$, $\varepsilon_0 \mu_0 c^2 = 1$,)

$$U = \frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}), \qquad \mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{3}$$

The electromagnetic momentum density $\mathbf{G}$ and the corresponding momentum flux tensor $\mathbf{T}$ are given by

$$\mathbf{G} = \mathbf{D} \times \mathbf{B}, \qquad \mathbf{T} = \mathbf{1}U - (\mathbf{DE} + \mathbf{BH}). \tag{4}$$

Show that

$$\mathrm{Tr}(\mathbf{G} \times \mathbf{1} \times \mathbf{S}) = a(\mathbf{G} \cdot \mathbf{S}), \tag{5}$$

where a is a number. Find a.

4. (**20 points.**) Friends A and B live on Earth. B takes a trip to Mars on a space ship that travels with a uniform speed $v < c$. They stay in touch using phones (of year 2023 make).

   (a) In the rest frame of A draw the world lines representing the position of A, position of B, and the phone signals that bounce off their phones, during the trip.

   (b) Repeat the above in the rest frame of B.