Homework No. 02 (Spring 2023)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Jan 31, 4.30pm

1. (**20 points.**) Consider the dyadic construction

$$\mathbf{T} = \mathbf{E}\,\mathbf{B} \tag{1}$$

built using the vector fields,

$$\mathbf{E} = E_x\,\hat{\mathbf{x}} + E_y\,\hat{\mathbf{y}}, \tag{2a}$$
$$\mathbf{B} = B\,\hat{\mathbf{y}}. \tag{2b}$$

Evaluate the following components of the dyadic:

$$\hat{\mathbf{x}}\cdot\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \hat{\mathbf{x}}\cdot\mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \hat{\mathbf{x}}\cdot\mathbf{T}\cdot\hat{\mathbf{z}} = \tag{3a}$$
$$\hat{\mathbf{y}}\cdot\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \hat{\mathbf{y}}\cdot\mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \hat{\mathbf{y}}\cdot\mathbf{T}\cdot\hat{\mathbf{z}} = \tag{3b}$$
$$\hat{\mathbf{z}}\cdot\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \hat{\mathbf{z}}\cdot\mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \hat{\mathbf{z}}\cdot\mathbf{T}\cdot\hat{\mathbf{z}} = \tag{3c}$$

Evaluate the scalars

$$\mathrm{Tr}(\mathbf{T}) = T_{ii}, \tag{4a}$$
$$\mathrm{Tr}(\mathbf{T}\cdot\mathbf{T}) = T_{ij}T_{ji}, \tag{4b}$$
$$\mathrm{Tr}(\mathbf{T}\cdot\mathbf{T}\cdot\mathbf{T}) = T_{ij}T_{jk}T_{ki}. \tag{4c}$$

Evaluate the following vector field constructions:

$$\hat{\mathbf{x}}\cdot\mathbf{T} = \qquad \hat{\mathbf{y}}\cdot\mathbf{T} = \qquad \hat{\mathbf{z}}\cdot\mathbf{T} = \tag{5a}$$
$$\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \mathbf{T}\cdot\hat{\mathbf{z}} = \tag{5b}$$
$$\hat{\mathbf{x}}\times\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \hat{\mathbf{y}}\times\mathbf{T}\cdot\hat{\mathbf{x}} = \qquad \hat{\mathbf{z}}\times\mathbf{T}\cdot\hat{\mathbf{x}} = \tag{5c}$$
$$\hat{\mathbf{x}}\times\mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \hat{\mathbf{y}}\times\mathbf{T}\cdot\hat{\mathbf{y}} = \qquad \hat{\mathbf{z}}\times\mathbf{T}\cdot\hat{\mathbf{y}} = \tag{5d}$$
$$\hat{\mathbf{x}}\times\mathbf{T}\cdot\hat{\mathbf{z}} = \qquad \hat{\mathbf{y}}\times\mathbf{T}\cdot\hat{\mathbf{z}} = \qquad \hat{\mathbf{z}}\times\mathbf{T}\cdot\hat{\mathbf{z}} = \tag{5e}$$
$$\hat{\mathbf{x}}\cdot\mathbf{T}\times\hat{\mathbf{x}} = \qquad \hat{\mathbf{x}}\cdot\mathbf{T}\times\hat{\mathbf{y}} = \qquad \hat{\mathbf{x}}\cdot\mathbf{T}\times\hat{\mathbf{z}} = \tag{5f}$$
$$\hat{\mathbf{y}}\cdot\mathbf{T}\times\hat{\mathbf{x}} = \qquad \hat{\mathbf{y}}\cdot\mathbf{T}\times\hat{\mathbf{y}} = \qquad \hat{\mathbf{y}}\cdot\mathbf{T}\times\hat{\mathbf{z}} = \tag{5g}$$
$$\hat{\mathbf{z}}\cdot\mathbf{T}\times\hat{\mathbf{x}} = \qquad \hat{\mathbf{z}}\cdot\mathbf{T}\times\hat{\mathbf{y}} = \qquad \hat{\mathbf{z}}\cdot\mathbf{T}\times\hat{\mathbf{z}} = \tag{5h}$$

2. (**20 points.**) Show that

$$\boldsymbol{\nabla}(\hat{\mathbf{r}}\cdot\mathbf{a}) = -\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{a})}{r} \tag{6}$$

for a uniform (homogeneous in space) vector $\mathbf{a}$.