Homework No. 04 (Spring 2023) PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University-Carbondale Due date: Thursday, 2022 Feb 16, 4.30pm

1. (20 points.) Let two conducting plates, with their inside faces occupying the y=0plane and y = a, consist of uniform positive and negative charge density σ flowing in opposite directions of $\hat{\mathbf{z}}$, respectively, described by drift velocity v_d such that the electric and magnetic field for this configuration is given by

$$\mathbf{E} = \begin{cases} \hat{\mathbf{y}} \frac{\sigma}{\varepsilon_0}, & \text{if } 0 < y < a, \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathbf{B} = \begin{cases} \hat{\mathbf{x}} \mu_0 \sigma v_d, & \text{if } 0 < y < a, \\ 0, & \text{otherwise.} \end{cases}$$
(1a)

$$\mathbf{B} = \begin{cases} \hat{\mathbf{x}} \,\mu_0 \sigma v_d, & \text{if } 0 < y < a, \\ 0, & \text{otherwise.} \end{cases}$$
 (1b)

Note,

$$cB = \beta_d E, \tag{2}$$

where $\beta_d = v_d/c$.

- (a) Derive the expressions in Eqs. (1) using Gauss's law and Ampère's law, respectively.
- (b) Explore the configuration in the rest frame of the flow associated with the drift velocity.
- (c) Evaluate the electromagnetic stress (force per unit area) on the plate at y=0. Consider the limiting case of $v_d = 0$ and match it with the results in the lecture of 2023 February 7.
- (d) (This Item is not for assessment.) Discuss the relativistic transformation of the stress.