Homework No. 06 (Spring 2023)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2023 Mar 7, 4.30pm

1. (**20 points.**) Lorentz transformation relates the energy E and momentum $\mathbf{p}$ of a particle when measured in different frames. For example, for the special case when the relative velocity and the velocity of the particle are parallel we have

$$\begin{pmatrix} E'/c \\ p' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E/c \\ p \end{pmatrix}. \tag{1}$$

   Photons are massless spin 1 particles whose energy and momentum are $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$, such that $\omega = kc$. Thus, derive the relativistic Doppler effect formula

$$\omega' = \omega\sqrt{\frac{1+\beta}{1-\beta}}. \tag{2}$$

   Contrast the above formula with the Doppler effect formula for sound.

2. (**20 points.**) Neutral π meson decays into two photons. That is,

$$\pi^0 \to \gamma_1 + \gamma_2. \tag{3}$$

   Energy-momentum conservation for the decay in the laboratory frame, in which the meson is not necessarily at rest, is given by

$$p_\pi^\alpha = p_1^\alpha + p_2^\alpha. \tag{4}$$

   Or, more specifically,

$$\left(\frac{E_\pi}{c}, \mathbf{p}\right) = \left(\frac{E_1}{c}, \mathbf{p}_1\right) + \left(\frac{E_2}{c}, \mathbf{p}_2\right), \tag{5}$$

   where E_π and $\mathbf{p}$ are the energy and momentum of neutral π meson, and E_i's and $\mathbf{p}_i$'s are the energies and momentums of the photons. Thus, derive the relation

$$m_\pi^2 c^4 = 2E_1E_2(1 - \cos\theta), \tag{6}$$

   where m_π is the mass of neutral π meson, and θ is the angle between the directions of $\mathbf{p}_1$ and $\mathbf{p}_2$.