Homework No. 10 (Spring 2023)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Thursday, 2023 Apr 13, 4.30pm

1. (**20 points.**) A particle with charge q moves on the z-axis with constant speed v, $\beta = v/c$, such that the position of the particle is

$$\mathbf{r}(t) = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + vt\,\hat{\mathbf{k}}. \tag{1}$$

The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r},t) = \gamma\frac{q}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z-vt)\hat{\mathbf{k}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}, \tag{2a}$$

$$c\mathbf{B}(\mathbf{r},t) = \beta\gamma\frac{q}{4\pi\varepsilon_0}\frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}. \tag{2b}$$

Using a clear diagram illustrate the direction of the fields at position (x, y, z) relative to the position of the particle at time t.

2. (**20 points.**) A charge particle with charge q moves on the z-axis with constant speed v, $\beta = v/c$. The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r},t) = \gamma\frac{q}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z-vt)\hat{\mathbf{k}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}, \tag{3a}$$

$$c\mathbf{B}(\mathbf{r},t) = \beta\gamma\frac{q}{4\pi\varepsilon_0}\frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}. \tag{3b}$$

Evaluate the electromagnetic momentum density for this configuration by evaluating

$$\mathbf{G}(\mathbf{r},t) = \varepsilon_0\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t) \tag{4}$$

and the flux of electromagnetic energy density for this configuration by evaluating

$$\mathbf{S}(\mathbf{r},t) = \varepsilon_0 c^2\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t). \tag{5}$$

3. (**20 points.**) A charge particle with charge q moves on the z-axis with constant speed v, $\beta = v/c$. The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r},t) = \gamma\frac{q}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z-vt)\hat{\mathbf{k}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}, \tag{6a}$$

$$c\mathbf{B}(\mathbf{r},t) = \beta\gamma\frac{q}{4\pi\varepsilon_0}\frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{\left[(x^2+y^2)+\gamma^2(z-vt)^2\right]^{\frac{3}{2}}}. \tag{6b}$$

Evaluate the electromagnetic field invariants

$$\mathbf{E}(\mathbf{r},t)^2 - c^2\mathbf{B}(\mathbf{r},t)^2 = \left(\frac{q}{4\pi\varepsilon_0}\frac{1}{[(x^2+y^2)+\gamma^2(z-vt)^2]}\right)^2 \tag{7}$$

and

$$\mathbf{E}(\mathbf{r},t)\cdot c\mathbf{B}(\mathbf{r},t) = 0. \tag{8}$$