

Homework No. 12 (Spring 2023)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2023 Apr 25, 4.30pm

1. **(20 points.)** An electron of charge e and mass m moves in a nearly circular orbit under the Coulomb forces produced by a proton. Suppose, as it radiates, the electron continues to move on a circle of ever decreasing radii.

- (a) The equation of motion for the electron given by Newton's laws of motion is

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}, \quad (1)$$

where the acceleration of the electron is the centripetal acceleration

$$a = \frac{v^2}{r}. \quad (2)$$

The total energy of the system E is the sum of the kinetic energy and electrostatic potential energy. Show that

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}. \quad (3)$$

- (b) A charge that is accelerating will lose energy in the form of radiation. The Larmor formula

$$P = -\frac{dE}{dt} = \left(\frac{\mu_0 c}{4\pi}\right) \frac{2}{3} \frac{e^2}{c^2} a^2, \quad (4)$$

gives the rate of loss of energy, the power P .

- (c) Combine the equation of motion of the electron with the Larmor formula to construct the following differential equation for the radius r ,

$$\frac{1}{c} \frac{dr}{dt} = -\frac{4}{3} \frac{r_0^2}{r^2}, \quad (5)$$

where $r_0 \sim 3 \times 10^{-15}$ m is the classical radius of the electron defined using the equality

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_0} = mc^2, \quad (6)$$

that is, when the electrostatic interaction energy is sufficient to create an electron. Solve this differential equation. In a finite time the electron reaches the center. Calculate how long it takes for the electron to hit the proton if it starts from an initial radius $a_0 \sim 0.5 \times 10^{-10}$ m, the Bohr radius. This is the classical lifetime of a Bohr atom.

- (d) Most atoms have lifetimes greater than the age of the universe, which is about 10^{17} s. This instability was one of the reasons for the discovery of quantum mechanics.