Solutions

PHYS-205A-002 (Midterm Exam 03) Fall 2023

Problem 1

$$\Delta K = W_1 + W_2 + \cdots = W_{total}$$

$$W_{total} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2}(25)(10.)^2 - \frac{1}{2}(25)(5.0)^2$$

$$= 940 \text{ J}$$

$$v_f^2 = \vec{v}_f \cdot \vec{v}_f$$

$$v_f = \sqrt{6.0^2 + 8.0^2} = 10. \text{ m/s}$$

$$v_i = \sqrt{4.0^2 + 3.0^2} = 5.0 \frac{\text{m}}{\text{s}}$$

Problem 2

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_f$$

$$\Delta K + \Delta U_g = W_N + W_F \quad (\text{since } W_N = 0)$$

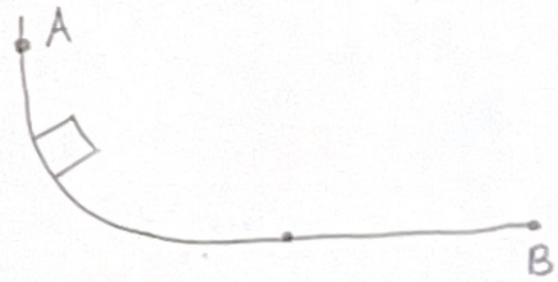
$$W_F = \Delta K + \Delta U_g$$

$$= \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 + mgh_B - mgh_A$$

$$= \frac{1}{2}(25)(4.0)^2 - 0 + 0 - (25)(9.8)(3.0)$$

$$= 200 - 735$$

$$= -540 \text{ J}$$



Problem 3.

When the work done by a force is independent of the path, it is called a conservative force.

Example: Gravity, Spring.

Problem 4

$$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) V_{2i}$$

$$= 0 + V_{2i} \quad (\text{for } m_1 = m_2)$$

$$= 0$$

$$V_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) V_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) V_{2i}$$

$$= V_{1i} + 0 \quad (\text{for } m_1 = m_2)$$

$$= 5.0 \ \frac{m}{s}$$

Problem 5

$W_N = 0$ (normal force is perpendicular to displacement in this configuration).

$$\Delta K + \Delta U^g + \Delta U^s = 0$$

$$K_A + U_A^g + U_A^s = K_c + U_c^g + U_c^s$$

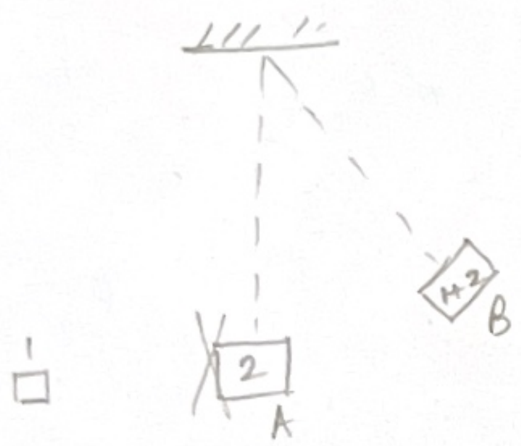
$$0 + mgh_A + \frac{1}{2} k x_A^2 = 0 + mgh_c + \frac{1}{2} k x_c^2$$

$$0 + (20.0)(9.8)(1.0) + 0 = 0 + 0 + \frac{1}{2}(2.0 \times 10^4) x_c^2$$

$$x_c = \sqrt{\frac{2(20.0)(9.8)(1.0)}{2.0 \times 10^4}} = 14 \times 10^{-2} \text{ m}$$

$$= 14 \text{ cm}$$

Problem 6



$$m_1 V_{1i} + m_2 \cancel{V_{2i}}^{=0} = (m_1 + m_2) V_f.$$

$$V_{1i} = \frac{m_1 + m_2}{m_1} V_f = \frac{3.030}{0.03} V_f = 101\, V_f.$$

$V_A = V_f$

$$\frac{1}{2}(m_1+m_2) V_A^2 + (m_1+m_2) g h_A = \frac{1}{2}(m_1+m_2) V_B^2 + (m_1+m_2) g h_B$$

$$V_A^2 = 2 g h_B$$

$$\left(\frac{V_{1i}}{101}\right)^2 = 2 g h_B$$

$$V_{1i} = 101 \sqrt{2 g h_B}$$

$$= 101 \sqrt{2(9.8)(0.300)} = 250 \frac{m}{s}$$

Problem 7

$$x_{cm} = \frac{\int dm\ x}{\int dm} = \frac{\int_0^L b x\, dx\ x}{\int_0^L b x\, dx}$$

$$\frac{dm}{dx} = bx$$

$$dm = b x\, dx.$$

$$= \frac{\int_0^L x^2 dx}{\int_0^L x\, dx}$$

$$= \frac{\frac{x^3}{3}\Big|_0^L}{\frac{x^2}{2}\Big|_0^L}$$

$$= \frac{\frac{L^3}{3} - 0}{\frac{L^2}{2} - 0} = \frac{2}{3} L$$

$$= 0.67\ m.$$