Equation Sheet for PHYS-205A University Physics

(Last updated: August 20, 2023)
This list will evolve during the semester.

1. General mathematics:

   (a) Units:

$$c = 10^{-2}, \quad m = 10^{-3}, \quad \mu = 10^{-6}, \quad n = 10^{-9}, \quad p = 10^{-12}. \tag{1a}$$
$$d = 10^{2}, \quad k = 10^{3}, \quad M = 10^{6}, \quad G = 10^{9}, \quad T = 10^{12}. \tag{1b}$$

   (b) Geometry of a right triangle:

$$\sin\theta = \frac{\text{opp. to angle}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opp. to angle}}{\text{adj to angle}}, \tag{2a}$$
$$\cos\theta = \frac{\text{adj. to angle}}{\text{hypotenuse}}, \qquad A^2 = A_x^2 + A_y^2. \tag{2b}$$

   (c) Quadratic equation:

$$a\,x^2 + b\,x + c = 0, \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{3}$$

   (d) Calculus:

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad \int x^n dx = \frac{x^{n+1}}{n+1}. \tag{4}$$

2. Kinematic equations:

   (a) Velocity and acceleration:

$$v(t) = \frac{dx}{dt}, \qquad a(t) = \frac{dv}{dt}. \tag{5}$$

   (b) Uniform velocity ($a = 0$):

$$\Delta x = v\Delta t. \tag{6}$$

   (c) Uniform acceleration:

$$v_f = v_i + a\,\Delta t; \qquad \Delta x = v_i\Delta t + \frac{1}{2}\,a\,\Delta t^2; \qquad v_f^2 = v_i^2 + 2\,a\,\Delta x; \tag{7a}$$
$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}; \qquad \Delta x = v_f\Delta t - \frac{1}{2}\,a\,\Delta t^2. \tag{7b}$$

   (d) Time of flight, horizontal range, and maximum height in projectile motion:

$$\Delta T = \frac{2v_0\sin\theta_0}{g}, \qquad R = \frac{v_0^2\sin 2\theta_0}{g}, \qquad H = \frac{v_0^2\sin^2\theta_0}{2g}. \tag{8}$$

   (e) Relative velocity: $\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AG} + \vec{\mathbf{v}}_{GB}$.

3. Forces:

   (a) Newton's law:

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = m\vec{\mathbf{a}} \tag{9}$$

   (b) Gravitational force:

$$F_G = \frac{Gm_1m_2}{R^2}, \qquad G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}. \tag{10}$$

   (c) Force due to friction:

$$F_\mathrm{f}\begin{cases} \le \mu_s N, & \text{(static case)},\\ = \mu_k N, & \text{(kinetic case)}. \end{cases} \tag{11}$$

   (d) Circular motion:

$$v = \omega r, \qquad \omega = 2\pi f, \qquad f = \frac{1}{T}, \tag{12}$$
$$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r = \frac{4\pi^2}{T^2}r \tag{13}$$

   (e) Resistive forces: $R = bv$ (for small speeds) and $R = \frac{1}{2}D\rho Av^2$ (for high speeds).

4. Work and energy:

   (a) Kinetic energy:

$$K = \frac{1}{2}mv^2 \tag{14}$$

   (b) Work done by a force:

$$W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{l}} \to Fd\cos\theta \tag{15}$$

   (c) Work-kinetic energy theorem:

$$W_1 + W_2 + \ldots = \Delta K \tag{16}$$

   (d) Potential energies:

$$U_g = mgh, \qquad U_s = \frac{1}{2}kx^2. \tag{17}$$

(e) Mechanical energy:

$$\Delta K + \Delta U_g + \Delta U_s = W_{\text{fric}} + \dots. \tag{18}$$

5. Linear momentum:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}, \qquad \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_i^f \vec{\mathbf{F}} dt. \tag{19}$$

(a) Conservation of linear momentum:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} + \ldots = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} + \dots. \tag{20}$$

(b) Elastic collision of two objects in one dimensional motion:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}, \tag{21a}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}. \tag{21b}$$

(c) Center of mass:

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} \to \frac{\int x\, dm}{\int dm}. \tag{22}$$

6. Rotational dynamics:

(a) Kinematic equations:

i. Constant angular speed ($\alpha = 0$): $\Delta\theta = \omega \Delta t$.

ii. Constant angular acceleration:

$$\omega_f = \omega_i + \alpha\, \Delta t, \qquad \Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\, \Delta t^2, \tag{23}$$

$$\omega_f^2 = \omega_i^2 + 2\,\alpha\, \Delta\theta, \qquad \Delta\theta = \omega_f \Delta t - \frac{1}{2}\alpha\, \Delta t^2, \qquad \frac{\Delta\theta}{\Delta t} = \frac{\omega_i + \omega_f}{2}. \tag{24}$$

(b) Rotational inertia (moment of inertia): $I = \int r^2\, dm$.

$$I = \begin{cases} MR^2, & \text{Point mass, distance } R \text{ from axis,} \\ MR^2, & \text{Circular ring, about symmetry axis of ring,} \\ \frac{2}{3}MR^2, & \text{Spherical shell, about diameter,} \\ \frac{1}{2}MR^2, & \text{Solid cylinder, about symmetry axis of cylinder,} \\ \frac{2}{5}MR^2, & \text{Solid sphere, about diameter.} \end{cases} \tag{25a}$$

(c) Torque:

$$\tau = RF \sin\theta, \qquad \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}. \tag{26}$$

(d) Rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2} I\omega^2. \tag{27}$$

(e) Angular momentum:

$$L = I\omega, \qquad \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \tag{28}$$

7. Gravitation:

$$\vec{\mathbf{F}} = -\hat{\mathbf{r}} \frac{G m_1 m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2. \tag{29}$$

$$U = -\frac{G m_1 m_2}{r}. \tag{30}$$