# Midterm Exam No. 01 (Fall 2023)

**PHYS 500A: MATHEMATICAL METHODS**

*School of Physics and Applied Physics, Southern Illinois University–Carbondale*

Date: 2023 Sep 29

1. (**20 points.**) Using the property of Kronecker $\delta$-symbol and Levi-Civita symbol evaluate the following using index notation,

$$(\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm})\delta_{jn}. \tag{1}$$

2. (**20 points.**) For a vector field $\mathbf{A}$, evaluate the vector identity

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{A}). \tag{2}$$

   Then, after the introduction of a scalar field $\psi$, evaluate

$$\boldsymbol{\nabla} \left[\psi \cdot (\boldsymbol{\nabla} \times \mathbf{A})\right]. \tag{3}$$

3. (**20 points.**) Consider the dyadic construction

$$\mathbf{M} = \hat{\mathbf{i}}\,\hat{\mathbf{j}} + \hat{\mathbf{j}}\,\hat{\mathbf{i}}, \tag{4}$$

   where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are orthonormal vectors. Evaluate

$$\mathrm{tr}(\mathbf{M}^{108}). \tag{5}$$

4. (**20 points.**) A uniformly charged infinitely thin disc of radius $R$ and total charge $Q$ is placed on the $z = h \neq 0$ plane such that the normal vector on the disc is along the $z$ axis and the center of the disc at the origin. Write down the charge density of the disc in terms of $\delta$-functions and Heaviside step functions.

5. (**20 points.**) Evaluate

$$(\mathbf{a} \times \boldsymbol{\nabla}) \cdot (\mathbf{r} \times \mathbf{b}), \tag{6}$$

   where $\mathbf{r}$ is the the coordinate vector and $\mathbf{a}$ and $\mathbf{a}$ are coordinate independent vectors.