

# Midterm Exam No. 02 (Fall 2023)

## PHYS 500A: MATHEMATICAL METHODS

*School of Physics and Applied Physics, Southern Illinois University–Carbondale*

Date: 2023 Nov 3

1. (20 points.) Find the roots of the equation

$$z^3 + i = 0. \quad (1)$$

Mark the points corresponding to the roots on the complex plane.

2. (20 points.) Find all the roots of the equation

$$e^z = 1. \quad (2)$$

Mark the points corresponding to the roots on the complex plane.

3. (20 points.) Evaluate the contour integral

$$I = \oint_c dz \frac{e^{i\frac{z}{2}}}{(z^2 - \pi^2)}, \quad (3)$$

where the contour  $c$  is a unit circle going counterclockwise with center at the origin.

4. (20 points.) Evaluate the integral

$$\frac{1}{\pi} \int_{-\infty}^{\infty} dx \frac{e^{-i\pi x}}{x^2 + 1}. \quad (4)$$

5. (20 points.) Evaluate the contour integral

$$I = \frac{1}{2\pi i} \oint_c \frac{dz}{z^2}, \quad (5)$$

where the contour  $c$  is a unit circle going counterclockwise with center at the origin.