Homework No. 04 (Fall 2023)

PHYS 500A: MATHEMATICAL METHODS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Friday, 2023 Sep 22, 4.30pm

1. (**20 points.**) Consider the matrix

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}. \tag{1}$$

   (a) Find all the eigenvalues of the matrix A.

   (b) Find the normalized eigenvectors associated with all the eigenvalues of matrix A. (Simplification is achieved by writing the trignometric functions in terms of half angles. $1 - \cos\theta = 2\sin^2\theta/2$, $1 + \cos\theta = 2\cos^2\theta/2$, $\sin\theta = 2\sin\theta/2\cos\theta/2$.)

   (c) Determine the matrix that diagonalizes the matrix A.

2. (**20 points.**) Construct the matrix

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}, \tag{2}$$

   where

$$\boldsymbol{\sigma} = \sigma_x \hat{\mathbf{i}} + \sigma_y \hat{\mathbf{j}} + \sigma_z \hat{\mathbf{k}}, \tag{3}$$

$$\hat{\mathbf{r}} = \sin\theta\cos\phi\hat{\mathbf{i}} + \sin\theta\sin\phi\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}}. \tag{4}$$

   Use the following representation of Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

   Find the eigenvalues of the matrix $\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}$.

3. (**20 points.**) The Pauli matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

   is written in the eigenbasis of

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{7}$$

   Write σ_x in the eigenbasis of

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{8}$$

   Note that this representation has the arbitraryness of the choice of phase in the eigenvectors.