(Preview of) Midterm Exam No. 01 (2025 Spring)

PHYS 510: CLASSICAL MECHANICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Date: 2025 Feb 20

1. (**20 points.**) On functional derivative. Not available in preview mode.

2. (**20 points.**) Describe the motion corresponding to the Hamiltonian

$$H(\mathbf{r},\mathbf{p}) = \frac{p^2}{2m} + \frac{1}{2}k(x^2 - y^2), \tag{1}$$

where $\mathbf{r} = \hat{\mathbf{i}}\,x + \hat{\mathbf{j}}\,y + \hat{\mathbf{k}}\,z$ is position $\mathbf{p}$ is the associated momentum, and m and k are constants. In particular, plot the trajectory of motion for the the initial conditions

$$\mathbf{r}(0) = \hat{\mathbf{i}}\,0 + \hat{\mathbf{j}}\,R + \hat{\mathbf{k}}\,0, \tag{2a}$$
$$\mathbf{v}(0) = \hat{\mathbf{i}}\,\omega R + \hat{\mathbf{j}}\,0 + \hat{\mathbf{k}}\,0, \tag{2b}$$

where $\omega = \sqrt{k/m}$ and R is a non-zero length.

3. (**20 points.**) On Legendre transformation and immediate consequences of the transformation. Not available in preview mode.

4. (**20 points.**) Given the Lagrangian

$$L_1(z,v) = \frac{1}{2}mv^2 - mgz, \tag{3}$$

find the equation of motion. Next, given another Lagrangian

$$L_2(z,v) = \frac{1}{2}mv^2 - mgz + bvz, \tag{4}$$

find the equation of motion. Analyze and justify.

5. (**20 points.**) A relativistic charged particle of charge q and mass m in the presence of a known electric and magnetic field is described by

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}. \tag{5}$$

   (a) Find the Lagrangian for this system, that implies the equation of motion of Eq. (5), to be

$$L(\mathbf{x},\mathbf{v},t) = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q\mathbf{v}\cdot\mathbf{A}, \tag{6}$$

   using Hamilton's principle of stationary action.

(b) Determine the canonical momentum for this system

(c) Determine the Hamiltonian $H(\mathbf{r}, \mathbf{p})$ for this system to be

$$H(\mathbf{x}, \mathbf{p}, t) = \sqrt{m^2c^4 + (\mathbf{p} - q\mathbf{A})^2 c^2} + q\phi. \tag{7}$$