Midterm Exam No. 01 (2026 Spring)

PHYS 510: CLASSICAL MECHANICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Date: 2026 Feb 12

1. (**20 points.**) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{1}$$

of the following functional,

$$F[u] = \int_a^b dx \, \frac{u}{\sqrt{1 + \left(\frac{du}{dx}\right)^2}}, \tag{2}$$

assuming no variation at the end points.

2. (**20 points.**) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{3}$$

of the following functional,

$$F[u] = \int_a^b dx \, \sqrt{1 + \left(\frac{d^2u}{dx^2}\right)^2}, \tag{4}$$

assuming no variation at the end points.

3. (**20 points.**) Let the distance between two points on a plane be characterized by the infinitesimal statement

$$ds^2 = 2dx^2 + 2dxdy + dy^2. \tag{5}$$

The geodesic is the extremal of the functional

$$l[y] = \int_{(x_1,y_1)}^{(x_2,y_2)} ds. \tag{6}$$

Find the geodesics. Recognize them.

4. (**20 points.**) Given

$$U = U(S, V) \tag{7}$$

and

$$dU = T\,dS - P\,dV \tag{8}$$

and

$$F = U - TS, \tag{9}$$

evaluate

$$\frac{\partial F}{\partial S}. \tag{10}$$

5. (**20 points.**) Discuss the hanging chain problem in the gravitation field of a planet given by

$$\frac{GMm}{r}. \tag{11}$$