Homework No. 06 (2026 Spring)

PHYS 510: CLASSICAL MECHANICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Mar 3, 4.30pm

1. (**20 points.**) A mass m slides down a frictionless ramp that is inclined at an angle θ with respect to the horizontal. See Fig. 1. Assume uniform gravity g in the vertical downward direction.
   (a) What is the equation of constraint.
   (b) In terms of a suitable dynamical variable write a Lagrangian that describes the motion of the mass.
   (c) Find the equations of motion from the Lagrangian.

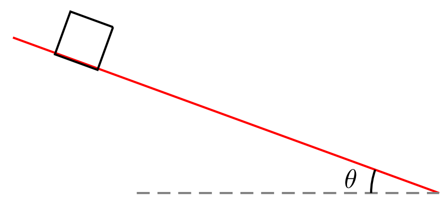

Figure 1: Problem 1.

2. (**20 points.**) The Atwood machine consists of two masses m_1 and m_2 connected by a massless (inextensible) string passing over a massless pulley. See Figure 2. Massless pulley implies that tension in the string on both sides of the pulley is the same, say T. Further, the string being inextensible implies that the magnitude of the accelerations of both the masses are the same. Let $m_2 > m_1$.
   (a) What is the constraint in the variables.
   (b) In terms of a suitable dynamical variable write a Lagrangian that describes the motion of the mass.
   (c) Find the equations of motion from the Lagrangian.

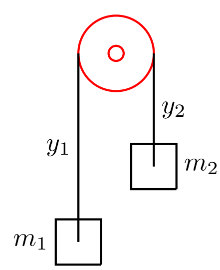

Figure 2: Problem 2.

3. (**20 points.**) [Based on Landau and Lifshitz. Section 7.] A particle of mass m moving with velocity $\mathbf{v}_1$ leaves a half-space in which the potential energy is a constant U_1 and enters another in which the potential energy is a different constant $U_2 > U_1$.

(a) Force is the manifestation of the system trying to attain minimum energy. Draw the velocity vector $\mathbf{v}_2$ in Fig. 3 that satisfies these conditions. Does it deflect away from normal or towards the normal?

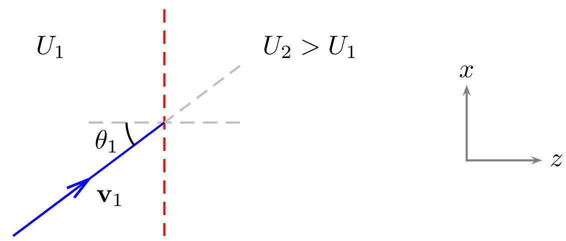

Figure 3: Problem 3.

(b) The potential energy can be described by

$$U(\mathbf{r}) = \begin{cases} U_1, & z < a, \\ U_2, & a < z. \end{cases} \tag{1}$$

In terms of the Heavyside step function

$$\theta(z) = \begin{cases} 0, & z < 0, \\ 1, & 0 < z, \end{cases} \tag{2}$$

show that the potential energy can be expressed in the form

$$U(\mathbf{r}) = U_1 + (U_2 - U_1)\theta(z - a). \tag{3}$$

(c) Show that a suitable Lagrangian for the motion is

$$L(\mathbf{r}, \mathbf{v}) = \frac{1}{2}mv^2 - U_1 - (U_2 - U_1)\theta(z - a). \tag{4}$$

Derive the relations

$$\frac{\partial L}{\partial \mathbf{v}} = m\mathbf{v}, \tag{5a}$$

$$\frac{\partial L}{\partial \mathbf{r}} = -\hat{\mathbf{z}}\,(U_2 - U_1)\,\delta(z - a). \tag{5b}$$

Recall that the derivative of Heaviside step function is a δ-function. Thus, derive the equation of motion

$$\frac{d}{dt}m\mathbf{v} = -\hat{\mathbf{z}}\,(U_2 - U_1)\,\delta(z - a). \tag{6}$$

(d) Show that the momentum in the plane perpendicular to $\hat{\mathbf{z}}$ is conserved. That is,

$$v_1 \sin\theta_1 = v_2 \sin\theta_2. \tag{7}$$

Show that the energy is conserved. That is,

$$\frac{1}{2}mv_1^2 + U_1 = \frac{1}{2}mv_2^2 + U_2. \tag{8}$$

Thus, derive the measure of deflection at the interface to be given by

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{1 - \frac{2(U_2 - U_1)}{mv_1^2}}. \tag{9}$$