Homework No. 04 (Spring 2026)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Thursday, 2026 Feb 12, 4.30pm

1. (**20 points.**) Given $\beta = v/c$, show that Lorentz transformation recovers Galilean ransformation for

$$\beta \to 0, \qquad c \to \infty, \qquad \text{such that} \qquad \beta c \to v. \tag{1}$$

   (a) Does Lorentz transformation recover Galilean transformation for $\beta \to 0$ alone?

   (b) Does Lorentz transformation recover Galilean transformation for $\beta \to 0$ and $c \to \infty$ alone?

2. (**20 points.**) A four-vector in the context of Lorentz tranformation can be described using the notation

$$a^\alpha = (a^0, a^1, a^2, a^3). \tag{2}$$

   Let

$$b^\alpha = (b^0, b^1, b^2, b^3) \tag{3}$$

   be another four-vector. The scalar product between two Lorentz vectors is given by

$$a^\alpha b_\alpha = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3. \tag{4}$$

   The square of the 'length' of the four-vector a^α is given by

$$a^\alpha a_\alpha, \tag{5}$$

   which is not necessarily positive. The length of a four-vector is invariant, that is, it is independent of the Lorentz frame. If two Lorentz four-vectors are orthogonal they satisfy

$$a^\alpha b_\alpha = 0. \tag{6}$$

   Orthogonality is an invariant concept.

   (a) Determine the length of

$$p^\alpha = (5, 0, 0, 3), \tag{7}$$

   where the numbers are in arbitrary units. Is it time-like, light-like, or space-like?

   (b) Find a four-vector of the form

$$q^\alpha = (q^0, 0, 0, q^3) \tag{8}$$

   that is perpendicular to p^α.

3. (**20 points.**) (Based on Hughston and Tod's book.) Prove the following.

   (a) If p^μ is a time-like vector and $p^\mu s_\mu = 0$ then s^μ is necessarily space-like.

   (b) If p^μ and q^μ are both time-like vectors and $p^\mu q_\mu < 0$ then either both are future-pointing or both are past-pointing.

   (c) If p^μ and q^μ are both light-like vectors and $p^\mu q_\mu = 0$ then p^μ and q^μ are proportional.

   (d) If p^μ is a light-like vector and $p^\mu s_\mu = 0$, then s^μ is space-like or p^μ and s^μ are proportional.

   (e) If u^α, v^α, and w^α, are time-like vectors with $u^\alpha v_\alpha < 0$ and $v^\alpha w_\alpha < 0$, then $w^\alpha u_\alpha < 0$.