Homework No. 06 (Spring 2026)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Mar 3, 4.30pm

1. (**20 points.**) A relativisitic particle in a uniform magnetic field is described by the equations

$$\frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v}, \tag{1a}$$

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}, \tag{1b}$$

where

$$E = mc^2\gamma, \tag{2a}$$

$$\mathbf{p} = m\mathbf{v}\gamma, \tag{2b}$$

and

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}. \tag{3}$$

Show that

$$\frac{d\gamma}{dt} = 0. \tag{4}$$

Then, derive

$$\frac{d\mathbf{v}}{dt} = \mathbf{v} \times \boldsymbol{\omega}_c, \tag{5}$$

where

$$\boldsymbol{\omega}_c = \frac{q\mathbf{B}}{m\gamma}. \tag{6}$$

Compare this relativistic motion to the associated non-relativistic motion.

2. (**20 points.**) A relativisitic particle in a uniform electric field is described by the equations

$$\frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v}, \tag{7a}$$

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}, \tag{7b}$$

where

$$E = mc^2\gamma, \tag{8a}$$

$$\mathbf{p} = m\mathbf{v}\gamma, \tag{8b}$$

and

$$\mathbf{F} = q\mathbf{E}. \tag{9}$$

Let us consider the configuration with the electric field in the $\hat{\mathbf{y}}$ direction,

$$\mathbf{E} = E\,\hat{\mathbf{y}}, \tag{10}$$

and initial conditions

$$\mathbf{v}(0) = 0\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}, \tag{11a}$$
$$\mathbf{x}(0) = 0\,\hat{\mathbf{x}} + y_0\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}. \tag{11b}$$

(a) In terms of the definition

$$\boldsymbol{\omega}_0 = \frac{1}{c}\frac{q\mathbf{E}}{m}, \tag{12}$$

show that the equations of motion are given by

$$\frac{d\gamma}{dt} = \boldsymbol{\omega}_0 \cdot \boldsymbol{\beta} \tag{13}$$

and

$$\frac{d}{dt}(\boldsymbol{\beta}\gamma) = \boldsymbol{\omega}_0. \tag{14}$$

(b) Since the particle starts from rest show that we have

$$\boldsymbol{\beta}\gamma = \boldsymbol{\omega}_0 t. \tag{15}$$

For our configuration this implies

$$\beta_x = 0, \tag{16a}$$
$$\beta_y\gamma = \omega_0 t, \tag{16b}$$
$$\beta_z = 0. \tag{16c}$$

Further, deduce

$$\beta_y = \frac{\omega_0 t}{\sqrt{1+\omega_0^2 t^2}}. \tag{17}$$

Integrate again and use the initial condition to show that the motion is described by

$$y - y_0 = \frac{c}{\bar{\omega}_0}\left[\sqrt{1+\bar{\omega}_0^2 t^2} - 1\right]. \tag{18}$$

Rewrite the solution in the form

$$\left(y - y_0 + \frac{c}{\omega_0}\right)^2 - c^2 t^2 = \frac{c^2}{\omega_0^2}. \tag{19}$$

This represents a hyperbola passing through $y = y_0$ at $t = 0$. If we choose the initial position $y_0 = c/\omega_0$ we have

$$y^2 - c^2 t^2 = y_0^2. \tag{20}$$

(c) The (constant) proper acceleration associated with this motion is

$$\alpha = \omega_0 c = \frac{c^2}{y_0}. \tag{21}$$

A Newtonian particle moving with constant acceleration α is described by equation of a parabola

$$y - y_0 = \frac{1}{2}\alpha t^2. \tag{22}$$

Show that the hyperbolic curve

$$y = y_0\sqrt{1 + \frac{c^2 t^2}{y_0^2}} \tag{23}$$

in regions that satisfy

$$\omega_0 t \ll 1 \tag{24}$$

is approximately the parabolic curve

$$y = y_0 + \frac{1}{2}\alpha t^2 + \ldots. \tag{25}$$