

Homework No. 07 (Spring 2026)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Mar 17, 4.30pm

1. **(20 points.)** Using Maxwell's equations, without introducing potentials, show that the electric and magnetic fields satisfy the inhomogeneous wave equations

$$\left(-\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \mathbf{E}(\mathbf{r}, t) = -\frac{1}{\epsilon_0} \nabla \rho(\mathbf{r}, t) - \frac{1}{\epsilon_0} \frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{J}(\mathbf{r}, t), \quad (1a)$$

$$\left(-\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \mathbf{B}(\mathbf{r}, t) = \mu_0 \nabla \times \mathbf{J}(\mathbf{r}, t). \quad (1b)$$

2. **(20 points.)** Consider the retarded Green's function

$$G(\mathbf{r} - \mathbf{r}', t - t') = \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} \delta\left(t - t' - \frac{1}{c}|\mathbf{r} - \mathbf{r}'|\right). \quad (2)$$

- (a) For $\mathbf{r}' = 0$ and $t' = 0$ show that

$$G(r, t) = \frac{1}{4\pi r} \delta\left(t - \frac{r}{c}\right). \quad (3)$$

- (b) Then, evaluate

$$\int_{-\infty}^{\infty} dt G(r, t). \quad (4)$$

- (c) From the answer above, what can you comment on the physical interpretation of $\int_{-\infty}^{\infty} dt G(r, t)$.

3. **(20 points.)** Evaluate the integral

$$\int_{-\infty}^{\infty} dx g(x) \delta(b^2 - a^2 x^2). \quad (5)$$

Hint: Use the identity

$$\delta(F(x)) = \sum_r \frac{\delta(x - a_r)}{\left|\frac{dF}{dx}\right|_{x=a_r}}, \quad (6)$$

where the sum on r runs over the roots a_r of the equation $F(x) = 0$.

4. (20 points.) Evaluate the integral

$$\zeta(s) = \lim_{\epsilon \rightarrow 0^+} \int_{\epsilon}^{\infty} dx \left(\frac{\pi}{x}\right)^s \delta(\sin x) \quad (7)$$

as a sum. The resultant sum is the Riemann zeta function. Determine $\zeta(2)$.
Hint: Use the identity

$$\delta(F(x)) = \sum_r \frac{\delta(x - a_r)}{\left| \frac{dF}{dx} \Big|_{x=a_r} \right|}, \quad (8)$$

where the sum on r runs over the roots a_r of the equation $F(x) = 0$.