

Homework No. 08 (Spring 2026)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Mar 24, 4.30pm

1. (20 points.) A particle with charge q moves on the z -axis with constant speed v , $\beta = v/c$, such that the position of the particle is

$$\mathbf{r}(t) = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + vt\hat{\mathbf{k}}. \quad (1)$$

The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r}, t) = \gamma \frac{q}{4\pi\epsilon_0} \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z - vt)\hat{\mathbf{k}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}, \quad (2a)$$

$$c\mathbf{B}(\mathbf{r}, t) = \beta\gamma \frac{q}{4\pi\epsilon_0} \frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}. \quad (2b)$$

Using a clear diagram illustrate the direction of the fields at position (x, y, z) relative to the position of the particle at time t .

2. (20 points.) A charge particle with charge q moves on the z -axis with constant speed v , $\beta = v/c$. The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r}, t) = \gamma \frac{q}{4\pi\epsilon_0} \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z - vt)\hat{\mathbf{k}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}, \quad (3a)$$

$$c\mathbf{B}(\mathbf{r}, t) = \beta\gamma \frac{q}{4\pi\epsilon_0} \frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}. \quad (3b)$$

Evaluate the electromagnetic momentum density for this configuration by evaluating

$$\mathbf{G}(\mathbf{r}, t) = \epsilon_0 \mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t) \quad (4)$$

and the flux of electromagnetic energy density for this configuration by evaluating

$$\mathbf{S}(\mathbf{r}, t) = \epsilon_0 c^2 \mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t). \quad (5)$$

3. (20 points.) A charge particle with charge q moves on the z -axis with constant speed v , $\beta = v/c$. The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r}, t) = \gamma \frac{q}{4\pi\epsilon_0} \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z - vt)\hat{\mathbf{k}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}, \quad (6a)$$

$$c\mathbf{B}(\mathbf{r}, t) = \beta\gamma \frac{q}{4\pi\epsilon_0} \frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{[(x^2 + y^2) + \gamma^2(z - vt)^2]^{\frac{3}{2}}}. \quad (6b)$$

Evaluate the electromagnetic field invariants

$$\mathbf{E}(\mathbf{r}, t)^2 - c^2 \mathbf{B}(\mathbf{r}, t)^2 = \left(\frac{q}{4\pi\epsilon_0} \frac{1}{[(x^2 + y^2) + \gamma^2(z - vt)^2]} \right)^2 \quad (7)$$

and

$$\mathbf{E}(\mathbf{r}, t) \cdot c\mathbf{B}(\mathbf{r}, t) = 0. \quad (8)$$