Homework No. 09 (Spring 2026)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Thursday, 2026 Apr 9, 4.30pm

1. (**20 points.**) A particle, of charge q and mass m, always moves with speed $v \ll c$.

   (a) Consider the case when it oscillates on the x-axis with frequency ω_0 and amplitude A given by

   $$\mathbf{r}_1(t) = \hat{\mathbf{x}} A \cos\omega_0 t. \tag{1}$$

   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r}, t)$, radiated magnetic field $\mathbf{B}(\mathbf{r}, t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (b) Next, consider the case when the particle moves on a circle described by

   $$\mathbf{r}_2(t) = \hat{\mathbf{x}} A \cos\omega_0 t + \hat{\mathbf{y}} A \sin\omega_0 t. \tag{2}$$

   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r}, t)$, radiated magnetic field $\mathbf{B}(\mathbf{r}, t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (c) Show that the radiated electric and magnetic field is additive, that is, it is the sum of two oscillators.

   (d) Show that the radiated power is not additive, but exhibits interference effects. Identify the interference term for the circular motion.

   (e) Find directions $\hat{\mathbf{r}}$ for which the interference term goes to zero.

2. (**20 points.**) An electron of charge e and mass m moves in a nearly circular orbit under the Coulomb forces produced by a proton. Suppose, as it radiates, the electron continues to move on a circle of ever decreasing radii.

   (a) The equation of motion for the electron given by Newton's laws of motion is

   $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}, \tag{3}$$

   where the acceleration of the election is the centripetal acceleration

   $$a = \frac{v^2}{r}. \tag{4}$$

   The total energy of the system E is the sum of the kinetic energy and electrostatic potential energy. Show that

   $$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}. \tag{5}$$

(b) A charge that is accelerating will loose energy in the form of radiation. The Larmor formula

$$P = -\frac{dE}{dt} = \left(\frac{\mu_0 c}{4\pi}\right) \frac{2}{3} \frac{e^2}{c^2} a^2, \tag{6}$$

gives the rate of loss of energy, the power P.

(c) Combine the equation of motion of the electron with the Larmor formula to contruct the following differential equation for the radius r,

$$\frac{1}{c} \frac{dr}{dt} = -\frac{4}{3} \frac{r_0^2}{r^2}, \tag{7}$$

where $r_0 \sim 3 \times 10^{-15}$ m is the classical radius of the electron defined using the equality

$$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_0} = mc^2, \tag{8}$$

that is, when the electrostatic interaction energy is sufficient to create an electron. Solve this differential equation. In a finite time the electron reaches the center. Calculate how long it takes for the electron to hit the proton if it starts from an initial radius $a_0 \sim 0.5 \times 10^{-10}$ m, the Bohr radius. This is the classical lifetime of a Bohr atom.

3. (**20 points.**) Consider the motion of three non-relativistic particles (speed v_i small compared to speed of light c, $v_i \ll c$,) of identical charges $q_i = q$ and identical masses $m_i = m$, $i = 1, 2, 3$. The radiated power by the individual particles are given by the expressions

$$P_i(t) = \left(\frac{\mu_0 c}{4\pi}\right) \frac{2q^2}{3c^2} \mathbf{a}_i^2(t_e), \tag{9}$$

where $\mathbf{a}_i(t_e)$ is the acceleration of the i-th particle at the time of emission

$$t_e = t - \frac{r}{c}. \tag{10}$$

Let the total contribution to radiated power from the three particles together be denoted by the subscript $(1 + 2 + 3)$. Consider the motion of three particles moving on a circle with same uniform speed while remaining at the vertices of a equilateral triangle at each moment. Find the total radiated power $P_{(1+2+3)}(t)$. (Hint: The centripetal acceleration is in the radial direction.)

4. (**20 points.**) Consider the motion of two non-relativisitic particles (speed v_i small compared to speed of light c, $v_i \ll c$,) of identical charges $q_i = q$ and identical masses $m_i = m$, $i = 1, 2$. The individual radiation fields $\mathbf{B}_i(\mathbf{r}, t)$ and $\mathbf{E}_i(\mathbf{r}, t)$, the angular distribution of

emitted power $f_i(\theta, \phi, t)$, and the total radiated power $P_i(t)$, are given by the expressions,

$$c\mathbf{B}_i(\mathbf{r}, t) = -\frac{\mu_0}{4\pi}\frac{q}{r}\,\hat{\mathbf{r}} \times \mathbf{a}_i(t_e), \tag{11a}$$

$$\mathbf{E}_i(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\frac{q}{r}\,\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}_i(t_e)), \tag{11b}$$

$$f_i(\theta, \phi, t) = \frac{dP_i}{d\Omega} = \frac{1}{4\pi}\left(\frac{\mu_0 c}{4\pi}\right)\frac{q^2}{c^2}\big[\hat{\mathbf{r}} \times \mathbf{a}_i(t_e)\big]^2, \tag{11c}$$

$$P_i(t) = \left(\frac{\mu_0 c}{4\pi}\right)\frac{2q^2}{3c^2}\mathbf{a}_i^2(t_e), \tag{11d}$$

where $\mathbf{a}_i(t_e)$ is the acceleration of the i-th particle at the time of emission

$$t_e = t - \frac{r}{c}. \tag{12}$$

Let the total contribution to a physical quantity from the two particles together be denoted by the subscript $(1+2)$.

(a) Show that

$$\mathbf{B}_{(1+2)}(\mathbf{r}, t) = \mathbf{B}_1(\mathbf{r}, t) + \mathbf{B}_2(\mathbf{r}, t), \tag{13a}$$

$$\mathbf{E}_{(1+2)}(\mathbf{r}, t) = \mathbf{E}_1(\mathbf{r}, t) + \mathbf{E}_2(\mathbf{r}, t), \tag{13b}$$

thus, conclude that radiation fields are additive.

(b) Show that, in general, the angular distribution of radiated power and total radiated power from the two particles together is not additive and exhibits interference,

$$f_{(1+2)}(\theta, \phi, t) = f_1(\theta, \phi, t) + f_2(\theta, \phi, t) + f_{12}(\theta, \phi, t), \tag{14}$$

where

$$f_{12}(\theta, \phi, t) = 2\frac{1}{4\pi}\left(\frac{\mu_0 c}{4\pi}\right)\frac{q^2}{c^2}\big[\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e) - (\hat{\mathbf{r}}\cdot\mathbf{a}_1(t_e))(\hat{\mathbf{r}}\cdot\mathbf{a}_2(t_e))\big], \tag{15}$$

and

$$P_{(1+2)}(t) = P_1(t) + P_2(t) + P_{12}(t), \tag{16}$$

where

$$P_{12}(t) = 2\frac{1}{4\pi}\left(\frac{\mu_0 c}{4\pi}\right)\frac{2q^2}{3c^2}\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e). \tag{17}$$

Observe that the interference effect in the total radiated power is totally destructive for the case $\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e) = 0$. For this case, the interference effect in the angular distribution of radiated power is not necessarily destructive.

(c) Consider the motion of two particles moving on a circle with same uniform speed while remaining diametrically opposite to each other at each moment. Find the total radiated power $P_{(1+2)}(t)$. (Hint: The centripetal acceleration is in the radial direction.)

(d) Consider the motion of three particles moving on a circle with same uniform speed while remaining at the vertices of a equilateral triangle at each moment. Find the total radiated power $P_{(1+2+3)}(t)$.

(e) Find $P_{(1+2+3+4)}(t)$ for four particles moving on a circle such that they are at the vertices of a square at each moment.

(f) Find $P_{(1+\ldots+N)}(t)$ for N particles moving on a circle such that they are at the vertices of a N-sided polygon at each moment. Answer is zero.

(g) The quadrupole contribution will not be zero.