

Homework No. 01 (Fall 2026)

PHYS 500: MATHEMATICAL METHODS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Monday, 2026 Aug 25, 4.30pm

1. **(20 points.)** Verify the following relations:

$$\delta_{ij} = \delta_{ji}, \quad (1a)$$

$$\delta_{ii} = 3, \quad (1b)$$

$$\delta_{ik}\delta_{kj} = \delta_{ij}, \quad (1c)$$

$$\delta_{im}B_m = B_i, \quad (1d)$$

$$\varepsilon_{ijk} = -\varepsilon_{ikj} = \varepsilon_{kij}, \quad (1e)$$

$$\varepsilon_{iik} = 0, \quad (1f)$$

$$\delta_{ij}\varepsilon_{ijk} = 0. \quad (1g)$$

2. **(20 points.)** In three dimensions the Levi-Civita symbol is given in terms of the determinant of the Kronecker δ -functions,

$$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix} \quad (2a)$$

$$= \delta_{il}(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}) - \delta_{im}(\delta_{jl}\delta_{kn} - \delta_{jn}\delta_{kl}) + \delta_{in}(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}). \quad (2b)$$

Using the above identity show that

$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}, \quad (3a)$$

$$\varepsilon_{ijk}\varepsilon_{ijn} = 2\delta_{kn}, \quad (3b)$$

$$\varepsilon_{ijk}\varepsilon_{ijk} = 6. \quad (3c)$$

3. **(20 points.)** Given that the rectangular components of vector $\mathbf{A}$, represented by A_i , and the rectangular components of vector $\mathbf{B}$, represented by B_i , satisfy the commutation relations

$$[A_i, B_j] = i\hbar \delta_{ij}, \quad (4)$$

$\hbar = h/(2\pi)$, where h is the Planck constant. Using index notation and the properties of Kronecker δ -function express the left hand side of the following equation in the form on the right hand side,

$$\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A} = i\hbar \alpha. \quad (5)$$

Find α . Investigate the case when $\hbar = 0$.

4. **(20 points.)** Given that the rectangular components of vector $\mathbf{L}$, represented by L_i , satisfy the commutation relations

$$[L_i, L_j] = i\hbar \varepsilon_{ijk} L_k, \quad (6)$$

$\hbar = h/(2\pi)$, where h is the Planck constant. Using index notation and the properties of Kronecker δ -function and Levi-Civita symbol, show that

$$\mathbf{L} \times \mathbf{L} = i\hbar \mathbf{L}. \quad (7)$$

Investigate the case when $\hbar = 0$.

5. **(20 points.)** Given that the rectangular components of vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, represented by A_i , B_i , and C_i , respectively, satisfy the commutation relations

$$[A_i, B_j] = i\hbar \delta_{ij}, \quad (8a)$$

$$[B_i, C_j] = 0, \quad (8b)$$

$$[C_i, A_j] = 0, \quad (8c)$$

$\hbar = h/(2\pi)$, where h is the Planck constant. Using index notation and the properties of Kronecker δ -function and Levi-Civita symbol, show that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \alpha \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \beta \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) + i\hbar \gamma \mathbf{C}. \quad (9)$$

Find α , β , and γ . Investigate the case when $\hbar = 0$.

6. **(20 points.)** Given that the rectangular components of vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$, represented by A_i , B_i , C_i , and D_i , respectively, satisfy the commutation relations

$$[A_i, B_j] = i\hbar \delta_{ij}, \quad [B_i, C_j] = i\hbar \delta_{ij}, \quad (10a)$$

$$[A_i, C_j] = i\hbar \delta_{ij}, \quad [B_i, D_j] = i\hbar \delta_{ij}, \quad (10b)$$

$$[A_i, D_j] = i\hbar \delta_{ij}, \quad [C_i, D_j] = i\hbar \delta_{ij}, \quad (10c)$$

$\hbar = h/(2\pi)$, where h is the Planck constant. Using index notation and the properties of Kronecker δ -function and Levi-Civita symbol, show that

$$(\mathbf{A} \times \mathbf{B})(\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) - i\hbar \mathbf{A} \cdot (\mathbf{B} + \mathbf{C} - \mathbf{D}). \quad (11)$$

Investigate the case when $\hbar = 0$.