Homework No. 01 (Fall 2026)

PHYS 520A: ELECTROMAGNETIC THEORY I

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Aug 25, 4.30pm

1. (**20 points.**) Consider an infinitely thin uniformly charged ring of radius a and total charge Q. Let it be placed on the x-y plane with its center at the origin. Show that the charge density for the ring in spherical coordinates can be expressed in the form

$$\rho(\mathbf{r}') = \frac{Q}{2\pi a}\frac{\delta\left(\theta' - \frac{\pi}{2}\right)}{r'}\delta(r' - a). \tag{1}$$

   Verify that

$$\int d^3r'\rho(\mathbf{r}') = Q. \tag{2}$$

2. (**20 points.**) Consider a uniformly charged circular disc of radius a and total charge Q. Let the disc be infinitely thin. Let it be placed on the x-y plane with its center at the origin. Show that the charge density for the disc in spherical coordinates can be expressed in the form

$$\rho(\mathbf{r}') = \frac{Q}{\pi a^2}\frac{\delta\left(\theta' - \frac{\pi}{2}\right)}{r'}\theta(a - r'). \tag{3}$$

   Verify that

$$\int d^3r'\rho(\mathbf{r}') = Q. \tag{4}$$

3. (**20 points.**) (Based on Griffiths 3rd/4th ed., Problem 4.9.)

   (a) The electric field of a point charge q at distance $\mathbf{r}$ is

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{r}}{r^3}. \tag{5}$$

   The force on a point dipole in the presence of an electric field is

$$\mathbf{F} = (\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{E}. \tag{6}$$

   Use these to find the force on a point dipole due to a point charge.

   (b) The electric field of a point dipole $\mathbf{d}$ at distance $\mathbf{r}$ from the dipole is given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3\,\hat{\mathbf{r}}\,(\mathbf{d}\cdot\hat{\mathbf{r}}) - \mathbf{d}\right]. \tag{7}$$

   The force on a point charge in the presence of an electric field is

$$\mathbf{F} = q\mathbf{E}. \tag{8}$$

   Use these to find the force on a point charge due to a point dipole.

(c) Confirm that above two forces are equal in magnitude and opposite in direction, as per Newton's third law.

4. (**20 points.**) (Based on Griffiths 3rd/4th ed., Problem 4.8.)
The electric potential of a point dipole $\mathbf{d}$ placed at the origin is

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}. \tag{9}$$

Using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi \tag{10}$$

show that the electric field of a point dipole $\mathbf{d}$ at distance $\mathbf{r}$ from the dipole is given by the expression

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\big[3\,\hat{\mathbf{r}}\,(\mathbf{d}\cdot\hat{\mathbf{r}}) - \mathbf{d}\big]. \tag{11}$$

The interaction energy of a point dipole $\mathbf{d}$ in the presence of an electric field is given by

$$U = -\mathbf{d}\cdot\mathbf{E}. \tag{12}$$

Further, the force between the two dipoles is given by

$$\mathbf{F} = -\boldsymbol{\nabla}U. \tag{13}$$

Use these expressions to derive

(a) the interaction energy between two point dipoles separated by distance $\mathbf{r}$ to be

$$U = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\big[\mathbf{d}_1\cdot\mathbf{d}_2 - 3\,(\mathbf{d}_1\cdot\hat{\mathbf{r}})(\mathbf{d}_2\cdot\hat{\mathbf{r}})\big]. \tag{14}$$

(b) the force between the two dipoles to be

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{3}{r^4}\big[(\mathbf{d}_1\cdot\mathbf{d}_2)\,\hat{\mathbf{r}} + (\mathbf{d}_1\cdot\hat{\mathbf{r}})\,\mathbf{d}_2 + (\mathbf{d}_2\cdot\hat{\mathbf{r}})\,\mathbf{d}_1 - 5\,(\mathbf{d}_1\cdot\hat{\mathbf{r}})(\mathbf{d}_2\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\big]. \tag{15}$$

(c) Are the forces central? That is, is the force in the direction of $\mathbf{r}$?

(d) Are the forces on the dipole equal in magnitude and opposite in direction? That is, do they satisfy Newton's third law?