Homework No. 02 (Fall 2026)

PHYS 520A: ELECTROMAGNETIC THEORY I

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Sep 8, 4.30pm

1. (**20 points.**) Consider a configuration of charges $q_1, q_2, q_3, \ldots$, at positions $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots$, and let $\mathbf{r}_0$ be an arbitrary point in space. Define the postion vector of the charges with respect to $\mathbf{r}_0$ to be

$$\mathbf{R}_i = \mathbf{r}_i - \mathbf{r}_0. \tag{1}$$

The elecric monopole moment, the electric dipole moment, and the electric quadrupole moment, of this configuration is given by

$$Q = q_1 + q_2 + q_3 + \ldots, \tag{2a}$$
$$\mathbf{d} = q_1\mathbf{R}_1 + q_2\mathbf{R}_2 + q_3\mathbf{R}_3 + \ldots, \tag{2b}$$
$$\mathbf{q} = q_1(3\mathbf{R}_1\mathbf{R}_1 - R_1^2\mathbf{1}) + q_2(3\mathbf{R}_2\mathbf{R}_2 - R_2^2\mathbf{1}) + q_3(3\mathbf{R}_3\mathbf{R}_3 - R_3^2\mathbf{1}) + \ldots, \tag{2c}$$

respectively. Evaluate the monopole moment, the dipole moment vector, and the quadrupole moment tensor of three identical charges, each having charge q, positioned on the x axis at a, $2a$, and $3a$, respectively. [Optional investigation that will not be used for assessment: Under what conditions are the above moments independent of $\mathbf{r}_0$?]

2. (**20 points.**) Given the quadrupole tensor

$$\mathbf{q} = q_1(3\mathbf{R}_1\mathbf{R}_1 - R_1^2\mathbf{1}) + q_2(3\mathbf{R}_2\mathbf{R}_2 - R_2^2\mathbf{1}) + q_3(3\mathbf{R}_3\mathbf{R}_3 - R_3^2\mathbf{1}) + \ldots, \tag{3}$$

show that

$$\operatorname{tr}\mathbf{q} = 0. \tag{4}$$

3. (**20 points.**) The monopole moment, the dipole moment, and the quadrupole moment, of a charge distribution $\rho(\mathbf{r})$ is given by

$$Q = \int d^3r\, \rho(\mathbf{r}), \tag{5a}$$
$$\mathbf{d} = \int d^3r\, \rho(\mathbf{r})\, \mathbf{r}, \tag{5b}$$
$$\mathbf{q} = \int d^3r\, \rho(\mathbf{r}) \Big[3\mathbf{r}\mathbf{r} - r^2\mathbf{1}\Big], \tag{5c}$$

respectively. Consider a charge distribution consisting of a single point charge. If it is placed at the origin calculate the monopole moment, dipole moment, and quadrupole moment, of the charge distribution. Repeat the calculation if the position of the point charge is $(a, 0, 0)$.

4. (**20 points.**) We have three charges q_1, q_2, and q_3, at positions $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, respectively. If the configuration has zero electric monopole moment and zero electric dipole moment, then show that the three charges are collinear. [Hint: Let the three charges be q_1, q_2, and q_3, and their positions be $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, respectively. Show that we can express $(\mathbf{r}_1 - \mathbf{r}_3) = a(\mathbf{r}_1 - \mathbf{r}_2)$ and $(\mathbf{r}_2 - \mathbf{r}_3) = b(\mathbf{r}_1 - \mathbf{r}_2)$. Find a and b.] Further, show that the electric quadrupole moment of the configuration is

$$\mathbf{q} = q_h \Big[3(\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{r}_1 - \mathbf{r}_2) - (\mathbf{r}_1 - \mathbf{r}_2) \cdot (\mathbf{r}_1 - \mathbf{r}_2)\mathbf{1} \Big]. \tag{6}$$

where q_h is the harmonic mean of q_1 and q_2 given by

$$\frac{1}{q_h} = \frac{1}{q_1} + \frac{1}{q_2}. \tag{7}$$

5. (**20 points.**) Electric field lines due to four positive charges of equal magnitude placed at the vertices of a square are drawn in Fig. 1. Using Fig. 1 as a guide, estimate the approximate coordinates (choosing the red dot as origin) of [all] the points where a test charge will not experience a force. Also, comment on the stability (or instability) of a test charge kept at these points. Justify your reasoning.

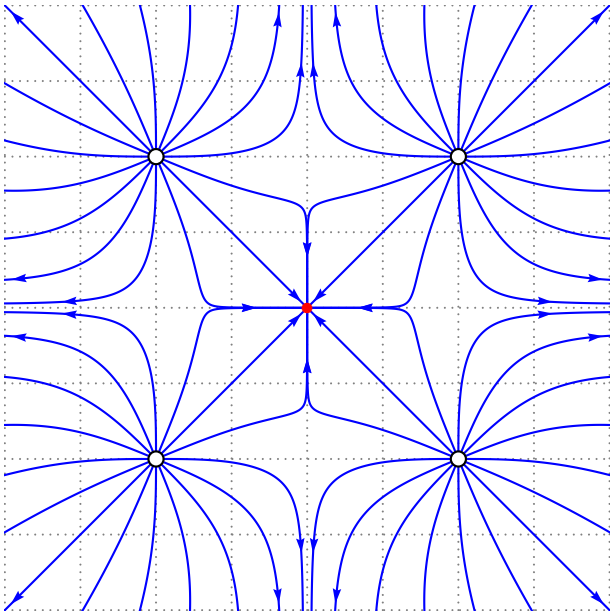

Figure 1: Electric field lines due to four identical positive charges placed at the vertices of a square.

6. (**20 points.**) Consider a configuration of nested concentric conducting spheres described by a charge density

$$\rho(\mathbf{r}) = \frac{Q}{4\pi a^2} \frac{5}{4} \left[\delta(r - a) - \delta\left(r - \frac{a}{2}\right) + \delta\left(r - \frac{a}{4}\right) - \delta\left(r - \frac{a}{8}\right) + \ldots \right]. \tag{8}$$

(a) Illustrate the configuration using a diagram.

(b) Find the total charge.

(c) Find the electric field in all regions.

(d) Draw the electric field lines. Be careful to satisfy all the constraints on electric field lines.

(e) Is it possible to rule out such an internal structure for an electron?