Homework No. 04 (Fall 2026)

PHYS 520A: ELECTROMAGNETIC THEORY I

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2026 Sep 22, 4.30pm

1. (**20 points.**) Show that the electric potential near an infinitely thin and infinitely long rod of uniform charge density λ can be expressed in terms of a distance L as

$$\phi(\mathbf{r}) = -\frac{2\lambda}{4\pi\varepsilon_0} \ln \frac{\rho}{2L}, \tag{1}$$

where ρ is the distance of the observation point from the rod. Show that the potential difference between two points is independent of L. The following is a guided exercise to derive the above expression.

(a) Consider a line segment of length $2L$ with uniform line charge density λ. When the rod is placed on the z-axis centered on the origin, show that the charge density can be expressed as

$$\rho(\mathbf{r}) = \lambda\delta(x)\delta(y)\theta(-L < z < L), \tag{2}$$

where $\theta(-L < z < L) = 1$, if $-L < z < L$ and $\theta(z) = 0$, otherwise.

(b) Inverting the Poisson equation for the electric potential, using the Green's function, or using the superposition principle for the electric potential, evaluate the electric potential for the rod as

$$\phi(\mathbf{r}) = \frac{\lambda}{4\pi\varepsilon_0}\left[\sinh^{-1}\left(\frac{L-z}{\sqrt{x^2+y^2}}\right) + \sinh^{-1}\left(\frac{L+z}{\sqrt{x^2+y^2}}\right)\right]. \tag{3}$$

(c) Using $\sinh t = (e^t - e^{-t})/2$, show that

$$\sinh^{-1} t = \ln(t + \sqrt{t^2+1}). \tag{4}$$

(d) Thus, express the electric potential of Eq. (3) in the form

$$\phi(\mathbf{r}) = \frac{\lambda}{4\pi\varepsilon_0}\left[-2\ln\frac{\rho}{L} + F\left(\frac{z}{L}, \frac{\rho}{L}\right)\right], \tag{5}$$

where $\rho^2 = x^2 + y^2$ and

$$F(a,b) = \ln[1 - a + \sqrt{(1-a)^2 + b^2}] + \ln[1 + a + \sqrt{(1+a)^2 + b^2}]. \tag{6}$$

(e) An infinite rod (on the z axis) is obtained by taking the limit $\rho \ll L, z \ll L$. Show that

$$\phi(\mathbf{r}) \xrightarrow{\rho\ll L, z\ll L} -\frac{2\lambda}{4\pi\varepsilon_0} \ln \frac{\rho}{2L}. \tag{7}$$

Hint: Series expand and keep only leading order terms.

(f) Using $\mathbf{E} = -\boldsymbol{\nabla}\phi$ determine the electric field for an infinite rod (placed on the z-axis) to be

$$\mathbf{E}(\mathbf{r}) = \frac{2\lambda}{4\pi\varepsilon_0} \frac{\hat{\boldsymbol{\rho}}}{\rho}. \tag{8}$$

2. (**20 points.**) Show that the electric potential near a flat conducting plate of infinite extent of uniform charge density (charg per unit area) σ can be expressed in terms of a distance R as

$$\phi(\mathbf{r}) = \frac{2\pi\sigma}{4\pi\varepsilon_0} \Big[R - |z| \Big], \tag{9}$$

where z is the distance of the observation point from the plate. Show that the potential difference between two points is independent of R. Using $\mathbf{E} = -\boldsymbol{\nabla}\phi$ determine the electric field for an infinite plate to be

$$\mathbf{E}(\mathbf{r}) = \frac{2\pi\sigma}{4\pi\varepsilon_0} \hat{\mathbf{n}}, \tag{10}$$

where $\hat{\mathbf{n}}$ is perpendicular to the plane.